Multibody Dynamics with Friction: Time-Stepping and Applications *

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In many engineered systems, nominally rigid bodies interact with each other through intermittent contact. Some of these contact are short-lived, as in the case of a typical collision, while others others persist for relatively long periods while sliding and/or rolling. Problems for which intermittent contact is of fundamental importance include trajectory planning for off-road vehicles (see Figure 1) and parts feeding for automated assembly (see Figure 2).

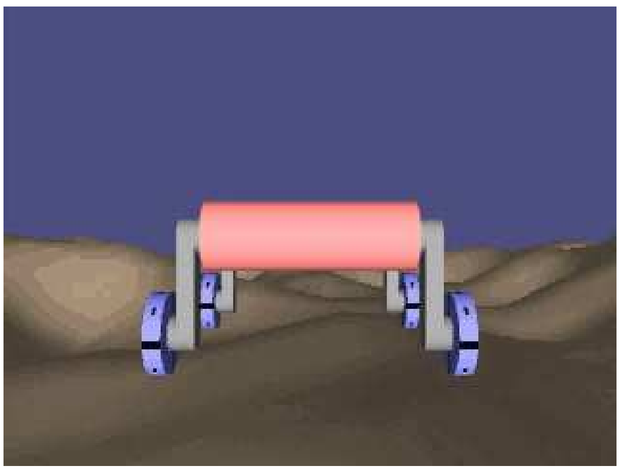

Figure 1: Four-wheeled, articulated vehicle on uneven terrain. For animations created in the Sandia software package Umbra, go to www.cs.sandia.gov/∼jctrink/rigid_body_dynamics.html

The analysis and simulation of multiple bodies in contact is quite difficult. Commercial mechanism analysis packages such as DADS, Working Model, and ADAMS have only recently begun to offer dynamic analysis of systems with contact, but currently the performance of these tools is unacceptable.

The fundamental problem from the perspective of analysis and simulation is that the underlying mathematical model is nonsmooth, due to the nonpenetration constraint on the bodies and the stick-slip behavior of dry friction. Commercial software software packages deal with the nonsmoothness by regularization techniques. These amount to penalty methods that allow small penetrations accompanied by large restoring forces generated by *ad hoc* lumped nonlinear springs. Stick-slip laws are relaxed similarly. The result, while smooth, is a stiff differential equation; the source of the problems mentioned above.

Our work on this problem begins with a formulation of multibody dynamics as a differential complementarity problem (DCP), which is a differential equation (the Newton-Euler dynamic equation) augmented with complementarity constraints

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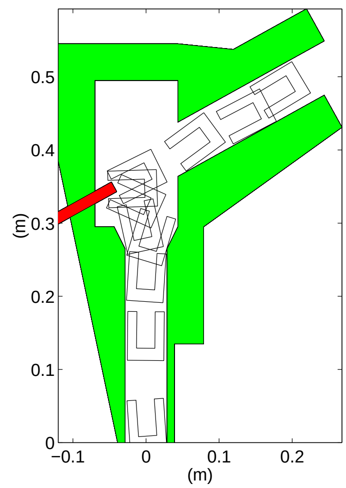

Figure 2: The exit orientation of the part must be with the closed end portion down, regardless of the entering orientation.

that rigorously enforce the nonpenetration constraint and stick-slip nature of dry friction, which are relaxed in commercial software. Complementarity constraints are of the following form:

$$\mathbf{0} \leq \boldsymbol{f} \perp \boldsymbol{z} \geq \mathbf{0},$$

where $\boldsymbol{f}$ and $\boldsymbol{z}$ are of length n, $\boldsymbol{f}$ is a given function of $\boldsymbol{z}$, $\perp$ is a short hand notation for $\boldsymbol{f}^T\boldsymbol{z} = 0$, and the inequalities apply element-by-element. The problem of finding solutions $\boldsymbol{z}$ satisfying the complementarity conditions is known as a complementarity problem.

Starting with the differential complementarity problem, we have derived implicit time-stepping methods. Instead of simply evaluating the right-hand side of a differential equation at each time step (as is done with penalty methods), our methods require the formulation and solution of a complementarity problem for each time step. While the computational cost for a single time step is higher in our methods, our methods allow larger time steps, are stable and accurate, and are faster overall. These methods have been incorporated into Sandia's simulation framework, Umbra.

DOD Applications and Future Work: DOD applications include the design of vehicles for semi-autonomous high-speed off-road travel and planning the disassembly of rubble heaps from collapsed building when one suspects that survivors may be trapped. Future work includes the development of methods for design optimization that formally treat parameter uncertainty. These methods will be based on sensitivity time-stepping methods for smooth differential equations and on ergodic theory.