Real-Time Generation of Fast Trajectories for Highly Maneuverable Underactuated Mechanical Systems

Kevin M. Lynch
Laboratory for Intelligent Mechanical Systems
Northwestern University
Evanston, IL 60208

Many underactuated mechanical systems can be written

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \left(\begin{array}{c} u \\ 0 \end{array} \right), \qquad (1)$$

perhaps after a feedback transformation, where $q \in \mathbb{R}^n$ is the configuration and $u \in \mathbb{R}^m$ is the control. These equations capture the dynamics of underactuated robot manipulators, spacecraft, ground vehicles, and underwater vehicles without drag. The $n-m$ underactuation constraints are the last $n-m$ rows of (1), written

$$M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + G_0(q) = 0. \qquad (2)$$

These are state-dependent constraints on the feasible accelerations $\ddot{q}$, and these constraints complicate the trajectory planning problem. The system may also be subject to a set of nonholonomic constraints of the form $\omega(q)\dot{q} = 0$.

The key to computationally efficient trajectory planning for such systems is to exploit the structure of the equations of motion. One such structure is differential flatness, which reduces the trajectory generation problem to curve fitting. The problem becomes significantly more complex in the presence of control and obstacle constraints, however.

We have recently discovered a class of underactuated mechanical systems we call **kinematically controllable** which permit obstacle and control constraints to be dealt with more naturally. For such systems, we can find a set of **decoupling velocity vector fields** on the the configuration space. These vector fields can be followed at any speed and acceleration without violating the constraints (2), and any configuration is reachable by following these vector fields. These decoupling vector fields together define a **kinematic reduction** of the dynamic system, and we can use methods from the literature on collision-free path planning for driftless kinematic systems. The resulting paths are time scaled to yield the time-optimal trajectory along the path. Trajectory planning is fast, because search occurs on the n-dimensional configuration space, not the $2n$-dimensional state space. Execution of a trajectory is fast, because it uses "natural" motions for the system.

In the absence of obstacles, path planning for some kinematically controllable systems reduces to simple closed-form inverse kinematics. This is often true for vehicle models, where the dynamics are invariant to group actions on $SE(n)$. Examples of such kinematically controllable systems are shown in Figures 1 and 2.

Future work will focus on (1) modifying existing kinematic path planners to suit kinematically controllable systems among obstacles and (2) feedback stabilization techniques for stabilizing planned trajectories.

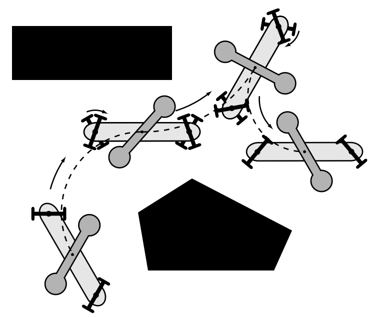

Figure 1: An obstacle-avoiding path for a snakeboard, which locomotes by steering the wheels and spinning the momentum rotor.

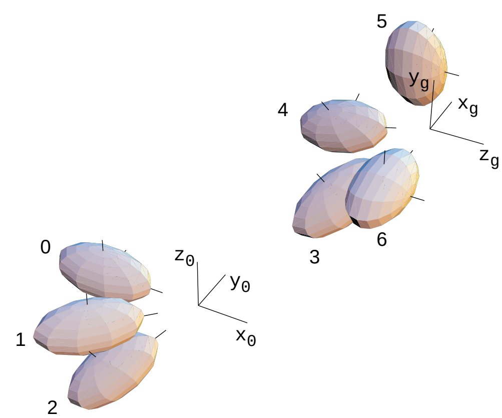

Figure 2: A trajectory for an autonomous underwater vehicle with only three body-fixed control forces.

[1] K. M. Lynch, N. Shiroma, H. Arai, and K. Tanie, "Collision-free trajectory planning for a 3-DOF robot with a passive joint," *International Journal of Robotics Research* 19(12):1171–1184, Dec 2000.
[2] F. Bullo and K. M. Lynch, "Kinematic controllability for decoupled trajectory planning of underactuated mechanical systems," *IEEE Transactions on Robotics and Automation* 17(4):402–412, Aug 2001.
[3] F. Bullo, A. D. Lewis, and K. M. Lynch, "Controllable kinematic reductions for mechanical systems: Concepts, computational tools, and examples," *2002 Mathematical Theory of Networks and Systems*.