Streaming Geometric Computations on the GPU

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Two Converging Trends in Computing …

- The accelerated development of graphics cards
  - developing faster than CPUs
  - GPUs are cheap and ubiquitous
- Increasing need for streaming computations
  - original motivation from dealing with large data sets
  - also interesting for multimedia applications, image processing, visualization etc.

What is a Stream?

- An ordered list of data items
- Each data item has the same type
  - like a tuple or record
- Length of stream is potentially very large
- Examples
  - *data records* in database applications
  - *vertex information* in computer graphics
  - *points, lines etc.* in computational geometry

Streaming Model

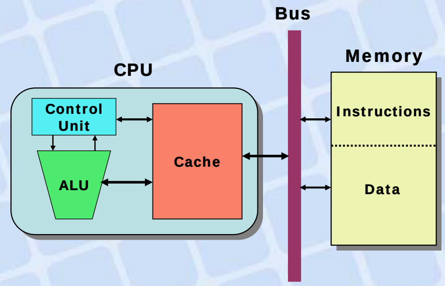
- Input presented as a sequence
- Algorithm works in a single pass
  - allowed one sequential scan over input
  - not permitted to move backwards mid-scan
- Workspace
  - typically o(n)
  - arbitrary computation allowed
- Algorithm efficiency
  - size of workspace and computation time

Streaming: Data driven to Performance driven

- Primary motivation is computing over *transient* data (data driven)
  - data over a network, sensor data, router data etc.
- Computing over *large, disk-resident* data which are expensive to access (data and performance driven)
- To improve algorithm performance

How does streaming help performance?

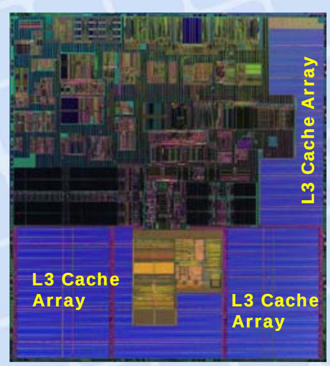
Von Neumann Bottleneck



Von Neumann Bottleneck

- Memory bottleneck
  - CPU processing faster than memory bandwidth
  - discrepancy getting worse
  - large caches and sophisticated prefetching strategies alleviate bottleneck to some extent
  - caches occupy large portions of real estate in modern ship design

Cache Real Estate



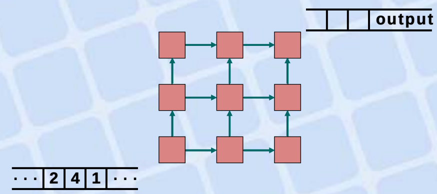
Die photograph of the Intel/HP IA-64 processor (Itanium2 chip)

Von Neumann Bottleneck

- Memory bottleneck
  - CPU processing faster than memory bandwidth
  - discrepancy getting worse
  - large caches and sophisticated prefetching strategies alleviate bottleneck to some extent
  - caches occupy large portions of real estate in modern ship design
- Unacceptable for computationally intensive applications

Proposed Solutions

- Systolic Arrays (Kung-Leiserson '78)
  - computational units arranged in specific topology (like grid or line)
  - data flows from one computational unit to its neighbors



Proposed Solutions

- Systolic Arrays (Kung-Leiserson '78)
  - computational units arranged in specific topology (like grid or line)
  - data flows from one computational unit to its neighbors
  - early graphics processor design based on systolic arrays

Proposed Solutions

- Systolic Arrays (Kung-Leiserson '78)
- SIMD Architectures
  - **single** set of instructions executed by different processors
  - **multiple** data streams fed to each processing unit
- Vector architectures
  - vector registers and vector processing units improve bandwidth

Stream Architectures

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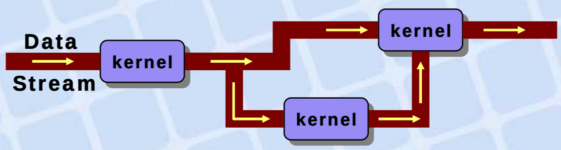
- All input in the form of streams
- Stream processed by a specialized computational unit called kernel
- Kernel performs the same operations on each stream element



Stream Architectures

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- Items processed in a FIFO fashion
- Reduced memory latency and cache requirements
- Simplified control flow
- Data-level parallelism
- Greater computational efficiency
- Examples
  - CHEOPS [Rixner et. al. '98] and Imagine [Kapasi et. al. '02]
  - high performance media applications

Graphics Pipeline: Rendering View

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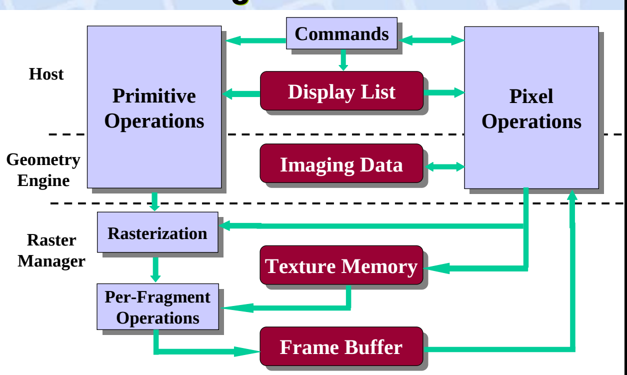

Graphics Hardware Pipeline

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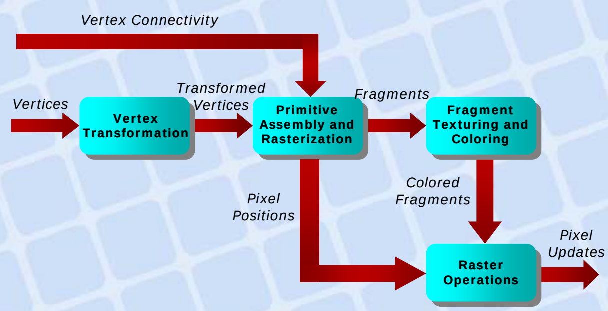
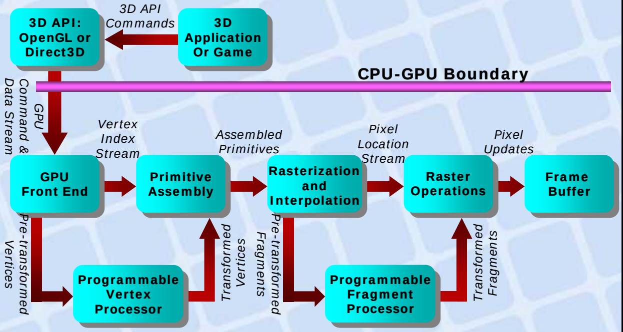


Courtesy: The Cg Tutorial [Fernando and Kilgard]

Programmable Graphics Pipeline

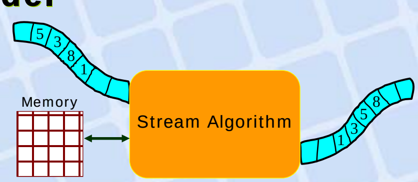


Courtesy: Cg Book [Fernando and Kilgard]

GPU: A Streaming Pipelined Architecture

- Inputs presented in streaming fashion
  - processed data items pass to next phase (1D systolic array?) and does not return
- Data-level parallelism
- Limited local storage
  - data items essentially carry their own state
- Pipelining: each item processed identically
- Not quite general purpose yet, but getting there

Standard Streaming Model



- What's the difference?
  - local memory (constant vs. *polylog* n)
  - pipelining restriction
  - multi-pass potential

GPU Capabilities

- Large instruction set for general purpose scalar and vector arithmetic
- General purpose memory access through textures
- Limited pointer indirection through dependent textures
- High level language support
  - Cg, HLSL

Diverse Applications

- Visibility, shadow computation
- Occlusion culling
- Motion planning, collision detection
- Physically-based modeling
- Image processing, FFT, wavelet analysis
- Radiosity, radiance, ray tracing
- Linear algebra, differential equations
- Computational geometry, solid modeling
- A lot more …

Streaming Geometric Computations on the GPU

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Geometric Algorithms in Hardware

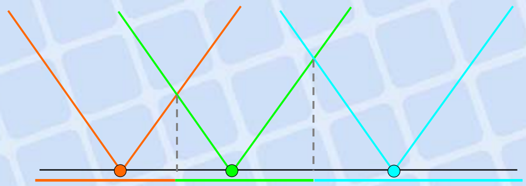
Use computational power of the GPU to implement geometric algorithms

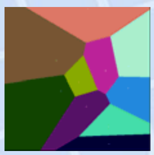
- Large speedups
- Circumvent problems of geometric complexity
- For some problems, CPU-bound solutions are hard
- Can handle other geometric settings (dynamic/kinetic)

Issues of Error

- Geometric input is continuous
- Computation and output are on finite-precision grid
- Error determined by grid resolution and rasterization process
- Approximation algorithms

Example: Voronoi Diagrams



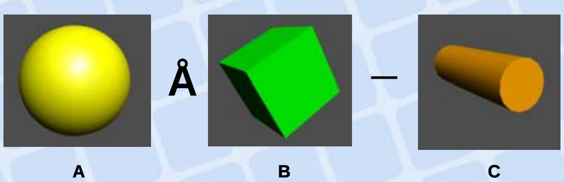


Hoff et. al.
Siggraph 99

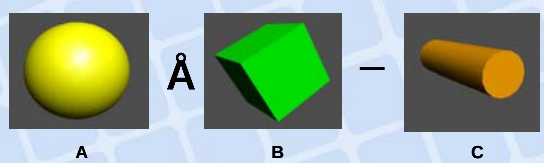
CSG Rendering on GPUs

Solid Modeling using CSG

- Constructive Solid Geometry (CSG)
  - way to model general solids
- Solid represented as Boolean combination of simple primitives



Solid Modeling using CSG



Solid Modeling using CSG

- Constructive Solid Geometry (CSG)
  - way to model general solids
- Solid represented as Boolean combination of simple primitives
- Two possibilities to render these objects
  - compute boundary representation using sophisticated geometric algorithms
    - expensive, robustness is an issue
  - directly render them implicitly
    - no mesh representation
    - effective for quick feedback during design process

Tree Normalization

- CSG expression
  - general expression with unions, intersections and differences
- Can be modified to canonical sum-of-products form
  - algorithm provided by Goldfeather et. al.
- Assumed as input in the rest of talk

Example: Bradley Fighting Vehicle

Courtesy: Army Research Lab

Bradley Fighting Vehicle

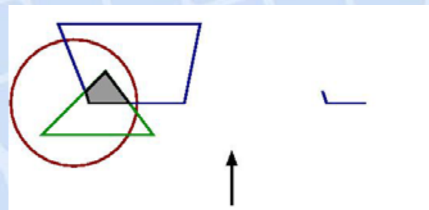
- Over 8000 primitive objects
  - polyhedra
  - ellipsoids, generalized quadrics
  - tori
  - surfaces of revolution
- Over 5000 CSG operations totally
  - individual trees vary from 10s to 100s of CSG operations

Previous Work

- Goldfeather et. al. ['85, '89]
  - use of parity
- Epstein et. al. '88, Rossignac & Wu '92
  - trickle algorithm
  - depth-interval buffers
- Weigand '95
  - implementation of Goldfeather on standard graphics pipeline
- Stewart et. al. ['98, '00, '02]
  - improvements on Goldfeather

Parity and Depth Test

- Idea introduced by Goldfeather et. al. ['85, '89]



- Requires n^2 rendering passes to compute intersection of n objects (single product)

Main Results

- Use two-sided depth test to sweep an arrangement of objects
- Perform CSG rendering in O(n) fewer passes (Guha et. al. – I3D '03)
  - optimal, no readbacks
- Extract arbitrary layer of a scene in *logarithmic* instead of *linear* passes
- First known lower bound results for algorithms on the GPU

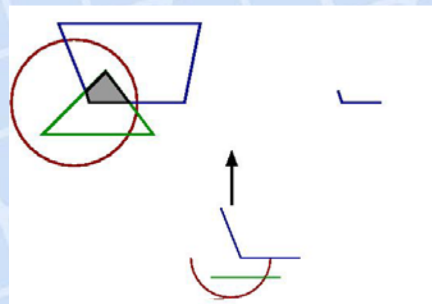
Two-Sided Depth Test

- Conceptualized by Mammen '89
- Implemented on current GPUs
  - shadow mapping hardware [Bastos, Everitt] on nVidia cards
  - simple fragment program
- Depth peeling applications
  - order-independent transparency
  - opacity light field mapping [Vlasik et. al. 03]

Algorithm (Single Product)

- Compute first level of arrangement
- Determine portions of level contained in intersection
  - depthmask = FALSE, depthtest = ·
  - Σ front faces – Σ back faces = n
  - use two-sided stencil test
- Advance to next level using depth peeling

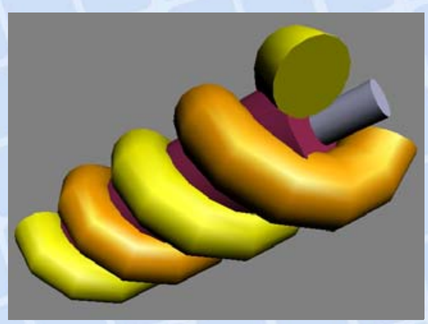
Example



Algorithm: Union of Products

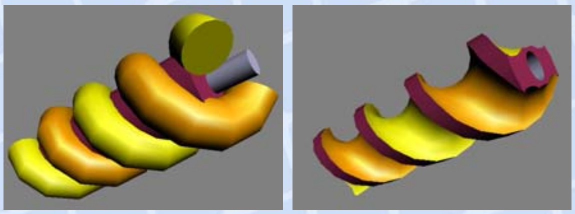
- Compute each product as before
- Merge depth and color field of current product with that of prefix sum
  - prefix depth stored in second depth texture
  - two pass merge step
- Rendering passes
  - single product: linear in product size
  - sum of products: sum of depth complexity

Example: Helix



Cyl1 – (Helix1 U Helix2 U Cyl2)

Example: Helix



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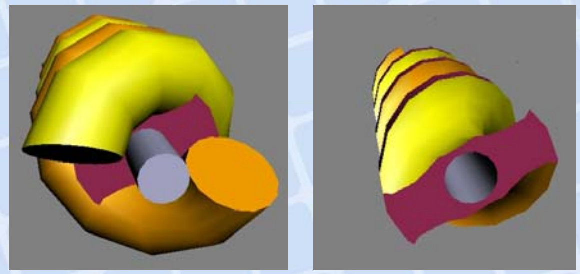
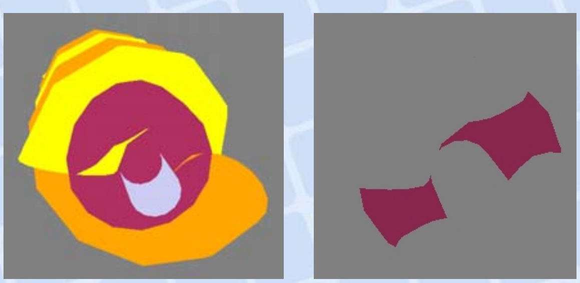
Example: Helix



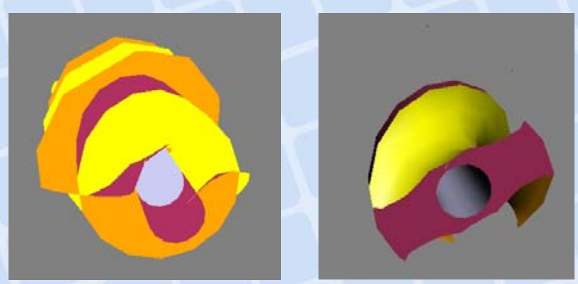
Cyl1 – (Helix1 U Helix2 U Cyl2)

Helix: First Layer

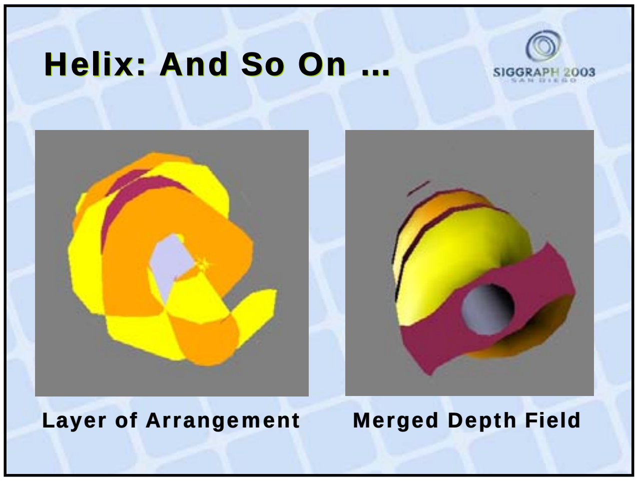


Layer of Arrangement | Resulting Depth Field

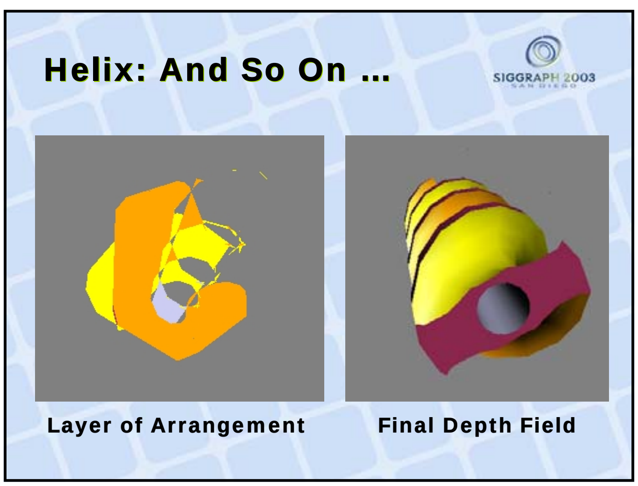
Helix: Second Layer

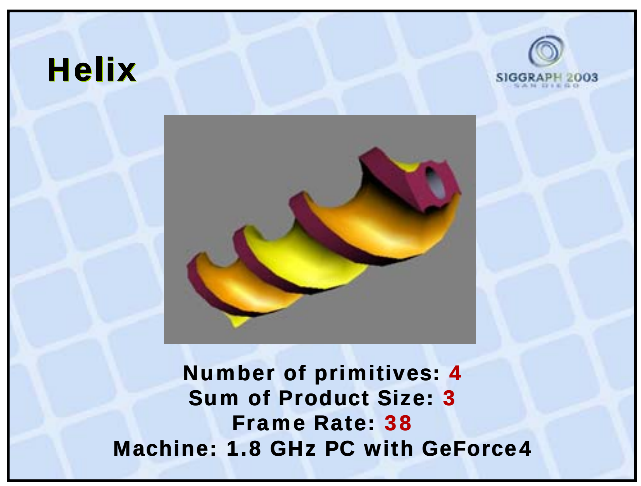


Layer of Arrangement | Merged Depth Field

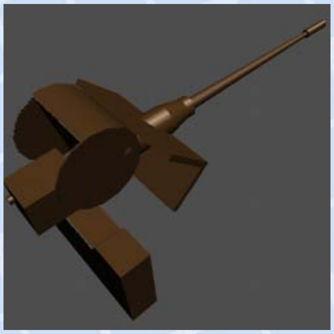

Helix: And So On …
SIGGRAPH 2003
Layer of Arrangement
Merged Depth Field


Helix: And So On …
SIGGRAPH 2003
Layer of Arrangement
Merged Depth Field


Helix: And So On …
SIGGRAPH 2003
Layer of Arrangement
Final Depth Field


Helix
SIGGRAPH 2003
Number of primitives: 4
Sum of Product Size: 3
Frame Rate: 38
Machine: 1.8 GHz PC with GeForce4

Bradley: 25 mm Gun



 # primitives: 64
SOP Size: 15
Frame Rate: 14

Bradley: Idlerwheel

 # primitives: 51
SOP Size: 10
Frame Rate: 12

Bradley: Drivewheel

 # primitives: 72
SOP Size: 30
Frame Rate: 2

Discussion

- Two-sided depth test
  - Novel algorithm for rendering CSG trees of general (but simple) closed shapes
    - optimal with respect to methods based on parity/counting
  - Arbitrary layer extraction from a scene
    - O(log n) rendering passes
    - planning under infeasible constraints
    - geometric optimization problems

Geometric Optimization on GPUs

What is Geometric Optimization?

- Computing statistical measures and approximate representations to geometric data
  - given a set of points, what is its diameter?
  - given a collection of triangles, find the smallest enclosing OBB?
  - given two intersecting convex polytopes, find the smallest translation vector of one to separate them – penetration depth?
  - given a collection of 2D shapes, pack them into smallest axis-aligned rectangle – polygon compaction?

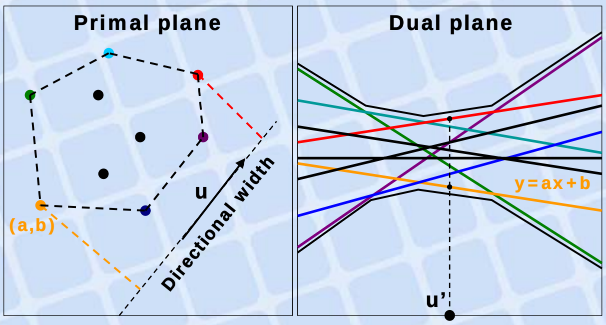
Geometric Optimization

- Solving exactly is computationally expensive
  - best fit OBB: $O(n^3)$
  - each problem needs specialized solution
- Still interesting from a streaming point of view
  - can we design algorithms that are efficient, yet provably approximate?
- Can GPUs be used to solve these problems?

Problem Characteristics

- Objective functions are (piecewise) algebraic
  - mostly linear
- Can be formulated as
  - lower/upper envelopes
  - overlay of multiple envelopes
- Hardware provides unified solutions to most of these problems
  - provably approximate solutions

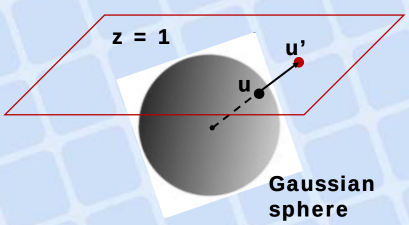
Point-Hyperplane Duality



Duality

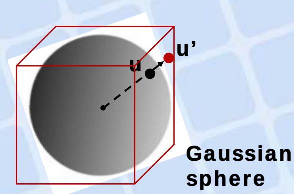
- Points in primal map to lines in dual
  - and vice versa
- Convex hull of points in primal
  - upper and lower envelope in dual
- Direction vector in primal
  - maps to point in the dual domain
  - central projection

Central Projection



Bounded Dual

- Cover space of directions with bounded cube



3D Diameter

- Diameter pair realized in the convex hull
- Dualize all the points
  - RGB space encodes point coordinates
- Upper and lower envelope determines antipodal pairs
- Two rendering passes to determine diameter
- Frame buffer resolution decides approximation factor

List of Problems Solved

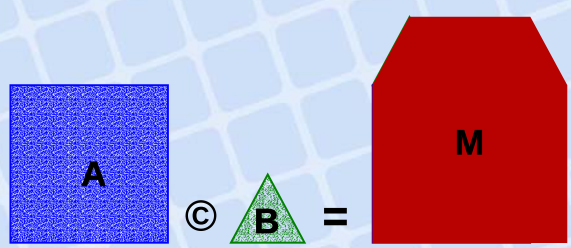
- Extent measures
  - 2D and 3D Width and Diameter
  - 2D and 3D Oriented Bounding Box
  - 1-Median, 1-Center and Closest pair
- Shape Matching/Fitting
  - Hausdorff and Summed-Hausdorff metrics under translation
  - best-fit line and circle
- Layered Manufacturing
- Path Planning (translation and rotation)

Penetration Depth

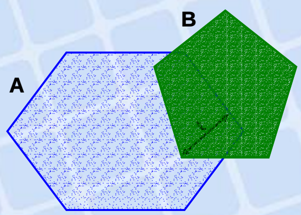
- Given two objects A and B, find smallest translation vector ***t*** such that (A + ***t***) is disjoint from B
- Equivalent to minimum distance from origin to (A © -B), the Minkowski sum
- Minkowski sum
  - quadratic complexity even for convex objects

Minkowski Sum

M = A © B = {a+b | a 2 A and b 2 B}



Penetration Depth



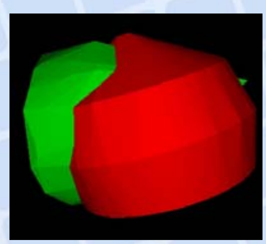
PD for Convex Objects

- Convex objects are closed under Minkowski sums
- Let M = A © -B
- Dualize all vertices of A and B
- Two observations
  - lower envelope of M's dual is sum of lower envelopes of A and B's dual
  - min. dist. from point ***p*** to M = min. dist. from ***p*** to all planes tangential to M

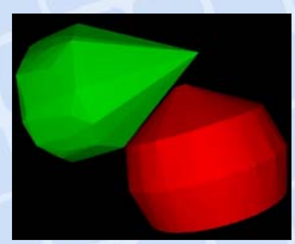
PD Algorithm

- Compute lower and upper envelope of dual planes to A and –B
- Sum corresponding envelopes
- Compute minimum
- Location of minimum in dual gives translation vector

PD Results

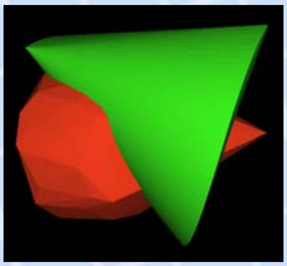


Original Position



After Separation

PD Results



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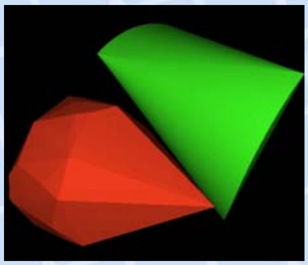

After Separation

Discussion

- Problem reformulation can lead to pipelined, streaming algorithms
- For many applications, back-end fast geometric computations are needed
- Resulting algorithms are very efficient
  - comparable, if not faster than sophisticated software techniques
- Current techniques restricted to 2D and 3D

Pipelined Streaming: Conclusions

- Stream architectures
  - alternate model for high-performance computing
  - GPU is readily accessible, easy-to-use platform for working with streams
    - numerous applications with demonstrable performance gain
  - strictly weaker than general streaming
    - probably stronger than circuit models

Open Issues

- Can we extend standard theoretical model of streaming to this somewhat restricted notion?
  - implications of multi-pass potential?
- What are limitation of stream architectures?
  - problems that can/cannot be addressed in this framework
- Issues of programming language design, compilers, OS and hardware design


Questions?
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