

Supplementary Material for A Multilevel SPH Solver with Unified Solid Boundary Handling

1. Algorithms

For implementation of our method, we give algorithms for our incompressible SPH solver (Algorithm 1), the PPE solve (Algorithm 2), and V-cycle (Algorithm 3).

Algorithm 1 Incompressible SPH Solver

- 1: **for all** particle i **do**
 - 2: find neighbor fluid and solid particles
 - 3: **for all** fluid particle i **do**
 - 4: compute density ρ_i
 - 5: **for all** fluid particle i **do**
 - 6: predict intermediate velocity \mathbf{u}_i^*
 - 7: **for all** fluid particle i **do**
 - 8: predict intermediate density ρ_i^*
 - 9: Solve the PPE (Algorithm 2)
 - 10: **for all** particle i **do**
 - 11: compute pressure force \mathbf{F}_i^p
 - 12: **for all** particle i **do**
 - 13: integrate velocity \mathbf{u}_i and position \mathbf{x}_i
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Algorithm 2 PPE Solve

- 1: **for all** fluid particle i **do**
 - 2: **if** i is Dirichlet or isolated particle **then**
 - 3: $p_i = 0$
 - 4: **else**
 - 5: **for all** neighbor solid particle s **do**
 - 6: compute $\frac{1}{\delta_s} \nabla W_{is}$
 - 7: compute $\alpha_i = \frac{\rho_0^2}{\rho_i^2} \|\sum_s \frac{1}{\delta_s} \nabla W_{is}\|^2$ and $b_i = \frac{\rho_i^* - \rho_0}{\Delta t^2}$
 - 8: **if** i is separated particle **then**
 - 9: $p_i = \max(0, \frac{b_i}{\alpha_i})$
 - 10: **else if** i is Poisson particle **then**
 - 11: **for all** neighbor fluid particle j **do**
 - 12: compute a_{ij}
 - 13: Assemble the matrix and source term (right hand side)
 - 14: Solve the PPE with our MGCG solver
 - 15: **for all** Poisson particle i **do**
 - 16: $p_i = \max(0, p_i)$
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Algorithm 3 V_cycle(l)

- 1: //r: residual
 - 2: //k^{pre} and k^{post}: the number of pre- and post-smoothing.
 - 3: **if** $l == 0$ **then**
 - 4: Solve $\mathbf{A}^0 \mathbf{p}^0 = \mathbf{b}^0$
 - 5: **else**
 - 6: **for** $k = 1$ to k^{pre} **do**
 - 7: PreSmooth($\mathbf{A}^l, \mathbf{b}^l, \mathbf{p}^l$)
 - 8: $\mathbf{r}^l = \mathbf{b}^l - \mathbf{A}^l \mathbf{p}^l$
 - 9: $\mathbf{b}^{l-1} = \text{Restrict}(\mathbf{r}^l)$
 - 10: $\mathbf{p}^{l-1} = 0$
 - 11: V_cycle($l - 1$)
 - 12: $\mathbf{p}^l = \mathbf{p}^l + \text{Interpolate}(\mathbf{p}^{l-1})$
 - 13: **for** $k = 1$ to k^{post} **do**
 - 14: PostSmooth($\mathbf{A}^l, \mathbf{b}^l, \mathbf{p}^l$)
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